AP® Calculus AB

Syllabus 1

Course Overview

My main objective in teaching AP® Calculus is to enable students to appreciate the beauty of calculus and receive a strong foundation that will give them the tools to succeed in future mathematics courses. Students know that they will work harder than ever, and our expectation is that this hard work will enable them to succeed in the course. We work together to help students discover the joys of calculus.

Primary Textbook

Hughes-Hallett, Deborah, et al. Calculus—*Single Variable*. 4th ed. New York: Wiley & Sons, 2005.

Course Planner **[C2]**

A Library of Functions (Chapter 1)

Students complete this review of precalculus materials over the summer.

Key Concept: The Derivative (Chapter 2)

3 weeks

1. How Do We Measure Speed?
2. Limits
3. The Derivative at a Point
4. The Derivative Function
5. Interpretations of the Derivative
6. The Second Derivative
7. Continuity and Differentiability

CBL Ball Toss Lab (see Student Activity 1, below) **[C3] [C5]**

Graphing the Derivative of a Function (see Student Activity 2) **[C4]**

Shortcuts to Differentiation (Chapter 3)

5 weeks

1. Powers and Polynomials

C2—The course teaches all topics associated with Functions, Graphs, and Limits; Derivatives; Integrals; and Polynomial Approximations and series as delineated in the Calculus Topic Outline in the AP Calculus Course Description.

C3—The course provides students with the opportunity to work with functions represented in a variety of ways—graphically, numerically, analytically, and verbally—and emphasizes the connections among these representations.

C5—The course teaches students how to use graphing calculators to help solve problems, experiment, interpret results, and support conclusions.

C4—The course teaches students how to communicate mathematics and explain solutions to problems both verbally and in written sentences.

2. The Exponential Function
3. The Product and Quotient Rules
4. The Chain Rule
5. The Trigonometric Functions
6. Applications of the Chain Rule and Related Rates
7. Implicit Functions
8. Linear Approximation and the Derivative
9. Using Local Linearity to Find Limits

Calculus in the Year 2000: New Ways of Teaching the Derivative and the Definite Integral, a three-hour video by Steve Olson

Tootsie Roll Pops Lab (see Student Activity 3)

"Investigating the Accuracy of the Tangent Line Approximation" (from the ancillary materials that accompany *Calculus—Single Variable*)

Using the Derivative (Chapter 4)

4 weeks

1. Using First and Second Derivatives
2. Families of Curves
3. Optimization and Modeling
4. Theorems About Continuous and Differentiable Functions

Optimization Project (see Student Activity 4)

Theorem of the Mean Police Man, a Mathematical Association of America Calculus Films video

Key Concept: The Definite Integral (Chapter 5)

3 weeks

1. How Do We Measure Distance Traveled?
2. The Definite Integral
3. Interpretations of the Definite Integral
4. Theorems About Definite Integrals

Using Calculus to Determine Distance Driven (see Student Activity 5)

Midterm Exam

Constructing Antiderivatives (Chapter 6)

3 weeks

1. Antiderivatives Graphically and Numerically
2. Constructing Antiderivatives Analytically
3. Differential Equations
4. Second Fundamental Theorem of Calculus
5. The Equations of Motion

Graphing the Antiderivative of a Function (similar to the activity on Graphing the Derivative of a Function from Chapter 2)

Integration (Chapter 7)

2 weeks

1. Integration by Substitution
2. Approximating Definite Integrals

Exploring Approximation Techniques Using a Calculator Program by Sam Gough et al., *Work Smarter Not Harder—Calculus Labs for TI-82 and TI-83.* **[C5]**

C5—The course teaches students how to use graphing calculators to help solve problems, experiment, interpret results, and support conclusions.

Differential Equations (Chapter 11)

4 weeks

1. What Is a Differential Equation?
2. Slope Fields
3. Separation of Variables
4. Growth and Decay
5. Applications and Modeling

Drawing Slope Fields (see Student Activity 6) **[C3] [C4] [C5]**

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Using the Definite Integral (Chapter 8)

3 weeks

1. Areas and Volumes

2. Applications to Geometry

Play-Doh Lab (see Student Activity 7)

Winplot Program (see Student Activity 7)

Volumes of Solids of Revolution—The Disk Method and Exploring Volume by Cross Sections (see Student Activity 7)

Teaching Strategies

On the first day of school, I begin with Chapter 2 (Key Concept: The Derivative). This sets the tone for the year. As discussed in the Student Activities section below, I start with a Calculator-Based Laboratory (CBL) ball toss experiment. Students explore the concept of average rate of change and discover the concept of instantaneous rate of change. Thereafter, student exploration and discovery continue to be an important aspect of the remaining topics. **[C5]**

C5—The course teaches students how to use graphing calculators to help solve problems, experiment, interpret results, and support conclusions

Throughout the course, students work together on a regular basis, both formally and informally. At times, I set up groups to work on a particular activity, but students do not need to be told to work together. Our classroom has tables instead of desks to make it more conducive to group work. When students are working on a problem, they will often work alone initially but then turn to their partners to collaborate. **[C4]**

C4—The course teaches students how to communicate mathematics and explain solutions to problems both verbally and in written sentences.

In discovering new concepts, the class works as a whole. It is not necessary for students to raise their hands. If students have a thought to share, they are welcome to make a contribution. If they are so inclined, students will go up to the board to illustrate a point. At times, I am able to step back and just listen to the interaction among my students as they explore a topic. **[C4]**

Technology can be used to help make calculus concepts come alive and it enables students to "see" what is being discussed. Students are issued TI-89 calculators. Our classroom also contains 10 computers and a SMART Board. We use *TI-Navigator*, *TI InterActive!*, and CBLs. Topics are presented using the "rule of four": graphically, numerically, algebraically, and verbally. Through this multifaceted approach, students gain an in-depth understanding of the material. **[C3] [C5]**

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Student Evaluation

Starting in October, I assign six AP free-response questions for students to work on for two weeks. Students may work on these questions with one other person and come to me for extra help. At the end of the two weeks, I randomly select one of these questions for a quiz. Students are graded as they would be graded on an AP Exam. (Free-response questions and scoring guidelines are available on AP Central®.)

When the second semester begins in February, we work on multiple-choice sections of AP Calculus Released Exams. Students have two weeks to complete a multiple-choice section. They may work on these questions with one other student and come to me for extra help. At the end of the two weeks, students hand in their work for grading. They are also quizzed on selected questions from the work they submit. The multiple-choice work alternates with the free-response work until the AP Exam administration.

Student Activities

1. On the first day of school, we begin the study of derivatives with a CBL experiment. Students toss a ball in the air and examine the height-versus-time graph generated. They fit the data with a quadratic equation for the position function to determine how high the ball went and how long it was in the air. Students compute the average velocity over a time interval. They are then asked to determine the velocity of the ball at exactly 0.06 seconds after the ball was tossed and explain how their answer was obtained. Finally, they zoom in on the graph of the position function near $t = 0.06$ until the graph looks like a line. Students find the slope of the line and compare it to their estimation of the instantaneous velocity at $t = 0.06$. **[C3] [C5]**

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2. When students are first learning about derivatives, I sketch the graph of an unknown function on the board. Each student comes up to the board and plots a point that would lie on the graph of the derivative of the function. We watch as the graph of the derivative unfolds. For example, I often graph a sine function on the board. At the maximum value of this function, the derivative would be zero. Students are anxious to plot these points. At the points of inflection, the derivative would be at a maximum or minimum. These are also easy points to plot. Gradually, students fill in the remaining values, and we can see that the derivative of a sine function is a cosine function. **[C4]**

C4—The course teaches students how to communicate mathematics and explain solutions to problems both verbally and in written sentences.

3. During our study of related rates, students suck on Tootsie Roll Pops to determine the rate of change of their radius; they then calculate the rate of change of the Pops' volume. Students measure the initial radius of a Pop with dental floss. They then suck on the Pop for 30 seconds, record its radius, suck for another 30 seconds, etc. They model the rate of change of the radius with some function of time. Students then use this rate of change to estimate the rate of change of the volume of the Pop when its radius is three-fourths of its original radius. This lab, "How Many Licks?," can be found in Ellen Kamischke's book, *A Watched Cup Never Cools*.

4. The study of optimization can be made more meaningful to students by asking them to design an optimum can. Students obtain a can of soda, soup, tuna, etc. They measure the height and diameter of the can and determine its volume. They then find the radius and height of the most cost-effective can that will hold the same volume and write an explanation, using well-written sentences, of the mathematics involved in making their determination. Students then construct the most cost-effective can, bring their original can and constructed can to school, and make a presentation to the class. (I obtained the idea for this activity, as well as the next one, from Christine Healy, a teacher at Bethpage High School in Bethpage, N.Y.)

5. After learning how to approximate a definite integral, students use these techniques to calculate the distance covered during a 20-minute drive with a friend or parent. Before beginning the drive, students record the car's odometer reading. Using the speedometer, they record the car's speed at one-minute intervals, noting any traffic conditions. At the end of the drive, they check the odometer reading again. Students then graph speed versus time and use integration techniques to approximate the distance traveled over the 20-minute interval. They compare this distance with the actual mileage determined by the odometer. Students are often amazed at the closeness of their approximation to the odometer reading. Students are to write a report on this project including an explanation on data collection, graphing of the data, interpretation of the data, and the closeness of their approximation to the odometer reading.

6. As an introduction to slope fields, I use an activity from the *AP Teacher's Guide to Accompany Calculus*—Single Variable. Using the graphing calculator screen with the grid turned on, I project a 3x3 grid onto the board and assign each student several coordinate points in the region (([1,1], [1,2]), etc.). **[C5]**

   For a given differential equation, each student computes the slope at his or her coordinate position and then goes to the board to draw a short line segment with the calculated slope and the coordinate point as the midpoint of the segment. For example, if $dy/dx = y$, the student with coordinates (1,1) would go to the board and at the point (1,1) draw a short line segment with a slope of 1. The student with coordinates (1,2) would go to the board and at the point (1,2) draw a small line segment with a slope of 2. (It is important that the second student draw a line segment whose slope is steeper than the slope of the first student's line segment.) Continuing in this fashion, the class would complete the slope field. At this point, all sorts of discussions can ensue. **[C3] [C4]**

7. One of the most difficult concepts for me as a student of calculus was finding the volume of solids. If students cannot visualize the solids, they have a more difficult time understanding how to compute the volume. To enable students to "see" the solids of revolution, I have purchased several "open-up" party decorations from a party supply store. Instead of just discussing how a line revolving around an axis forms a cone, students see the cone generated. (I found an open-up ice cream cone in one of the party stores.)

   I bring cans of Play-Doh into school and ask students to construct solids whose bases are bounded by two curves and whose cross sections are squares or equilateral triangles, etc. For example, students are given the graph of a circle and asked to construct a solid in which each cross section perpendicular to the base is an equilateral triangle. Students build the solids using Play-Doh and then use plastic knives or dental floss to cut through the solid and obtain the required cross sections.

   Students also use the *Winplot* program on the computer and see the solids come alive.

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We finish off this topic with two activities from *Work Smarter Not Harder*, a book of labs accompanied by a disk of calculator programs. Students can download a program onto their calculators that will enable them to enter a function, graph the function, and rotate the function about a line. The calculator will then display a cross section of the solid generated. Another program enables students to enter a function and display a cross section that is a square, isosceles right triangle, etc. By the time the students have completed these activities, they are quite comfortable with the topic.