

AP[®] Calculus AB

Syllabus 4

The following is an outline of the topics we cover and a typical sequence in which those topics are covered. The time spent is only an estimate of the average number of days allotted to the topic, because the actual time varies from year to year, depending upon the richness of class discussions that are generated. Also, with the wealth of interesting problems that are being supplied by those committed to reforming calculus, and with the always changing capabilities of technology, it is difficult to anticipate how many extra days a class might spend in exploration or discovery.

Each of my students has a graphing calculator of his or her own and is expected to have it in class each day. The calculator of choice for our mathematics department is the TI-83 Plus; however, some students own and use the TI-89. In my class, we use graphing calculators daily to explore, discover, and reinforce the concepts of calculus. Students may use the graphing calculators on some but not all assessments. [C5]

C5—The course teaches students how to use graphing calculators to help solve problems, experiment, interpret results, and support conclusions.

Pedagogical Issues

I encourage my students to explore and discover as much as possible. Now I do much less lecturing than I did in the past. I find that students are very comfortable with graphing calculators, and that *they* are asking the questions that I used to have to ask because they are “seeing” many of the concepts with the help of our investigations and, of course, their calculators. They sit in groups of three, four, or five to help each other by acting as mentors. [C4] [C5]

C4—The course teaches students how to communicate mathematics and explain solutions to problems both verbally, and in written sentences.

Sometimes I’ll gather two or three free-response questions given on previous AP Examinations for them to write up. Students are allowed to talk to each other about the problems, and may seek any additional help that is necessary. The emphasis is on students explaining and/or justifying their solutions to these problems orally. [C4]

These problems serve not only as a wonderful review opportunity but also as a way for students to discover weaknesses in concepts or in communication.

In addition, all student assignments require them to include justifications of their work in multiple representations, including symbolical and graphical representations that accompany complete written sentences explaining their processes and solutions. [C4]

AP[®] Calculus AB Course Outline [C2]

Local Linearity

(This replaces the early review chapter in most traditional texts.)

Time: Approximately 8 days

In this unit, we review graphs of basic families of functions, linear equations, and algebraic simplification, all in a context that is new to students and that leads them to the formal definition of the derivative.

1. Using the zoom and trace features of graphing calculators, we discover that almost all curves “straighten out” around a point $(a, f(a))$ when viewed close in. We write linear equations that seem to approximate the zoomed-in graph and then zoom out to discover a (nearly) tangent line. This is done with a variety of functions: polynomial, exponential, logarithmic, trigonometric, and piece-wise defined. [C3] [C5]
2. Using algebraic functions, a particular point $P(x_0, f(x_0))$ is selected, and then the slope of a secant chord joining P with $R(x, f(x))$ is simplified. The class investigates this slope value for various x -values, discovering what happens when x is very close in value to x_0 .
3. Using these same functions, the selected point becomes $P(a, f(a))$ and the general secant slope is discovered. The class then attempts to determine an expression that is likely to predict the secant slope value if x and a are extremely close in value.

C2—The course teaches all topics associated with Functions, Graphs, and Limits; Derivatives; Integrals; and Polynomial Approximations and Series as delineated in the Calculus BC Topic Outline in the *AP Calculus Course Description*.

C3—The course provides students with the opportunity to work with functions represented in a variety of ways—graphically, numerically, analytically, and verbally—and emphasizes the connections among these representations.

C5—The course teaches students how to use graphing calculators to help solve problems, experiment, interpret results, and support conclusions.

Functions and Limits

Time: Approximately 15 days

Note 1: I do *much* less formal work with limits than I used to, but I do want my students to understand why some limits exist and why others do not. We no longer do limit proofs, but we do spend time discussing why “closeness” is a relative term, and then look at the limit definitions to see how a mathematician resolves the issue. I also emphasize that with regard to

$$\lim_{x \rightarrow a} f(x) = L$$

what happens at $x = a$ is not relevant.

Note 2: The homework assignments given in this section also include problems that require the student to continue the investigation of the introductory work begun in our first unit. As limit notation is introduced, we incorporate that notation into our study of the limiting value of the secant segment’s slope as $x \rightarrow a$.

1. Review function notation, domain, and range.
2. Odd and even function definitions and graphical properties.

3. Introduction of limits intuitively.
4. Limit notation, including right- and left-hand limits.
5. Review of asymptotic behavior of rational and exponential functions using limit notation.
6. Estimating limits from graphs and from tables of values.
7. Calculating limits using algebra.
8. Presentation of a definition of $\lim_{x \rightarrow a} f(x) = L$ only to show students how a formal definition addresses the idea of “closeness” and how it excludes concern for what occurs when $x = a$. (Optional)
9. Continuity and graphical properties of continuous functions (including Intermediate Value Theorem and Extreme Value Theorem).
10. Investigating functions that are not continuous at $x = a$.

Note 3: Students will use the graphing calculator to estimate limits, determine the asymptotic behavior of a function, investigate the continuity of a function, and analyze the graphs of functions. [C5]

C5—The course teaches students how to use graphing calculators to help solve problems, experiment, interpret results, and support conclusions.

The Derivative and Applications of Differentiation

Time: Approximately 45 days

Note: Students find little new in the definition of the derivative since they have been working with it from day one. Time is spent, however, in looking at what occurs when a function is not differentiable at a point $(a, f(a))$. We explain the non-differentiability both graphically and by noting reasons why the conditions required by the definition fail in particular cases. Students will also use a graphing calculator to verify the non-differentiability of certain functions at a point. [C5]

1. Definition of the derivative

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

2. Instantaneous rate of change (as the limiting value of average rate of change)
3. Investigating functions differentiable at $x = a$ as well as those not differentiable at $x = a$ algebraically and graphically.
4. Using f' to investigate or confirm the increasing/decreasing behavior of f both algebraically and graphically.

5. The relationship between the graphs of f and f' . Given one graph, sketching the other.
6. Relative (local) extrema and the first derivative test.
7. Absolute extrema in the context of applied problems.
8. Derivatives of Algebraic Functions
 - a. The Power rule is usually “discovered” in the introductory unit and is now supported with proof.
 - b. The Sum, Constant Multiple, and Product rules are prompted by geometric arguments but are supported with proof.
9. Derivatives of circular functions
10. Review of composite functions
11. The Chain rule, prompted by guided investigation and then supported with proof.
12. Implicit Differentiation
13. Related Rates
14. The Second Derivative: concavity and points of inflection.
15. Making connections between f , f' , and f'' in tables and in graphs.
16. Local linearity revisited
 - a. Using linearization to approximate transcendental function values.
 - b. Using slope fields to better understand the significance of differential equations and perhaps to discover the general behavior of a function that is a solution to a differential equation by knowing an initial condition.
17. The relationship between differentiability and continuity.
18. The Mean Value Theorem (without proof)

The Integral and Applications of Integration

Time: Approximately 45 days

Note 1: Several days are spent in the first phase of this unit. Using graphing technology and available programs, we carefully explore Riemann sums. Although the investigation of area is the basic context, we include regions below the axis as well as those above. Students discover that Riemann sums are not always good predictors of the answer if area is the question. [C3] [C5]

When studying area, we discover that a Riemann sum is providing us with the sum of “signed” areas. The groundwork laid then carries over into applications involving distance as well, where a Riemann sum over an interval is able to provide displacement results but not necessarily total distance traveled.

C3—The course provides students with the opportunity to work with functions represented in a variety of ways—graphically, numerically, analytically, and verbally—and emphasizes the connections among these representations.

C5—The course teaches students how to use graphing calculators to help solve problems, experiment, interpret results, and support conclusions.

Note 2: Techniques of integration are de-emphasized. I want students to be able to find antiderivatives that are “reasonable,” but we get most of our practice by dealing with applied problems rather than by doing pages of drill problems that I used to assign. It is important for students to occasionally encounter functions whose antiderivatives cannot be found, or cannot easily be found. This encourages them to use numerical techniques or to intelligently use the capabilities of available technology.

1. Riemann sums (left-hand, right-hand, and midpoint sums only) to approximate the area of a region bounded by continuous functions.
2. Evaluating limits of Riemann sums over equal subdivisions to determine area of regions bounded by polynomial functions on the interval $[0, b]$.
3. The definite integral as a limit of Riemann sums.
4. The Fundamental Theorem of Calculus (with proof).
5. Indefinite integrals: antiderivatives of known functions and using simple substitutions.
6. Integration by parts.
7. Numerical approximations to definite integrals using tables and graphs.
 - a. Review Riemann sums.
 - b. Trapezoidal rule.
8. Using definite integrals whose integrands are velocity functions to show that accumulating rates of change in distance yields net distance traveled.
9. Rectilinear motion.
10. Volumes of known cross sections as limits of Riemann sums, including
 - a. Sums of discs
 - b. Sums of washers
 - c. Sums of cylindrical shells
 - d. Sums of other cross-sectional slices
11. Average value of a function.
12. Variable separable differential equations involving simple polynomial and trigonometric functions.

Transcendental Functions

Time: Approximately 30 days

1. Defining $\ln(x) = \int \frac{1}{t} dt$
2. Properties of natural logarithms
3. Logarithmic differentiation
4. Inverse functions and their derivatives
5. Exponential functions as inverses of logarithmic functions
6. The definition of e
7. Differentiation and Integration involving e^u , a^u , and $\log_a u$
8. Exponential growth and decay problems
9. Inverse trigonometric functions and their derivatives
 - a. Review of all previously studied concepts and applications using transcendental functions.

Major Textbook

Stewart, James. *Calculus*, 4th ed. Pacific Grove, Calif.: Brooks/Cole Publishing Company, 1999.