AP® Calculus AB

Syllabus 3

Course Design and Philosophy

Students do best when they have an understanding of the conceptual underpinnings of calculus. Rather than making the course a long laundry list of skills that students have to memorize, we stress the "why" behind the major ideas. If students can grasp the reasons for an idea or theorem, they can usually figure out how to apply it to the problem at hand. We explain to them that they will study four major ideas during the year: limits, derivatives, indefinite integrals, and definite integrals. As we develop the concepts, we explain how the mechanics go along with the topics.

Teaching Strategies

During the first few weeks, we spend extra time familiarizing students with their graphing calculators. Students are taught the rule of three: Ideas can be investigated analytically, graphically, numerically, and verbally. Students are expected to relate the various representations to each other. **[C3, C5]**

It is important for them to understand that graphs and tables are not sufficient to prove an idea. Verification always requires an analytic argument. Each chapter exam includes one or two questions that involve only graphs or numerical data.

I believe it is important to maintain a high level of student expectation. I have found that students will rise to the level that I expect of them. A teacher needs to have more confidence in the students than they have in themselves.

We also stress communication as a major goal of the course. Students are expected to explain problems using proper vocabulary and terms. Like many teachers, I have students explain solutions on the board to their classmates. This lets me know which students need extra help and which topics need additional reinforcement. Also, I have students explain and/or justify their solutions to problems in well-written sentences. **[C4]**

We often coordinate science activities using the Texas Instruments Calculator-Based Laboratory. Students will better understand the concepts of calculus when they see concrete applications.

Much of calculus depends on an understanding of a concept taught in a previous lesson. Students form study groups and tutor themselves. **[C4]**

C3—The course provides students with the opportunity to work with functions represented in a variety of ways—graphically, numerically, analytically, and verbally—and emphasizes the connections among these representations.

C5—The course teaches students how to use graphing calculators to help solve problems, experiment, interpret results, and support conclusions.

C4—The course teaches students how to communicate mathematics and explain solutions to problems both verbally and in written sentences.

Major Text

Finney, Ross L., Franklin D. Demana, Bert K. Waits, and Daniel Kennedy. *Calculus—Graphical, Numerical, Algebraic.* 1st ed. Menlo Park: Scott-Forseman Addison-Wesley, 1999.

Calculator Ideas

The graphing calculator is used to help students develop an intuitive feel for concepts before they are approached through typical algebraic techniques.

We use the calculator as a tool to illustrate ideas and make discoveries about functions in Calculus. The four required functionalities of graphing technology are:

1. Finding a root
2. Sketching a function in a specified window
3. Approximating the derivative at a point using numerical methods
4. Approximating the value of a definite integral using numerical methods

Students are also required to make connections between the graphs of functions and their analysis, and conclusions about the behavior of functions when using a graphing calculator. **[C5]**

C5—The course teaches students how to use graphing calculators to help solve problems, experiment, interpret results, and support conclusions.

Activities

The following sample activities demonstrate ways to help students gain an increased understanding of calculus.

Limits

If your calculator has a "table" feature, it can be used to zoom in on a limit numerically. **[C3, C5]**

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For example, to find

$$\lim_{x \to 2} \frac{x-2}{x^2-4}$$

we view the values of the function from x-values from 1.5 to 2.5 with an increment step of 0.1. At $x = 2$ the table records "error" or "not defined." Students should see that the y-values seem to follow a pattern. Redo the process beginning at 1.9 with a step size of 0.01, and observe that the y-values are converging to 0.25. The process can be repeated with smaller and smaller steps.

The limit can also be shown visually by graphing the function in a window that has a pixel step of 0.1. Trace the function beginning at $x = 1$. Each step shows the corresponding x- and y-coordinates, but at $x = 2$, the y-coordinate disappears. It "reappears" when the tracing continues at $x = 2.1$. Students can see graphically that the y-coordinates cluster at about 0.25 as x is near 2.

For comparison, do the same exploration with

$$\lim_{x \to 2} \frac{x^2+4}{x-2}$$

This function is also undefined at $x = 2$, but the y-values do not converge as x approaches 2. Instead, the values explode, giving students a numerical look at asymptotic behavior.

The Derivative of the Sine Function (This activity works well on an overhead display.)

Graph the function $y = \sin x$ in a standard trigonometric viewing window. Estimate the slope of the tangent line at various x-values and plot the slope values as a function of x on the overhead screen. **[C3, C5]** (The slope values are clearly zero at the turning points and can be estimated to be +1 or -1 at the x-intercepts. A few more estimates will enable students to guess the curve.) Students should see that the slope curve follows the path of the cosine function. To test this conjecture, graph the numerical derivative of the sine in the same window. Then graph the cosine function and note that the two graphs are superimposed. Tracing gives the same values on both curves. From this point it is easy to proceed to an analytic proof of

$$\frac{d}{dx}(\sin x) = \cos x$$

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AP Calculus AB Course Outline

Unit 1: Precalculus Review (2–3 weeks) [C2]

C2—The course teaches all topics associated with Functions, Graphs, and Limits; Derivatives; and Integrals as delineated in the Calculus AB Topic Outline in the *AP Calculus Course Description*.

A. Lines

1. Slope as rate of change
2. Parallel and perpendicular lines
3. Equations of lines

B. Functions and graphs

1. Functions
2. Domain and range
3. Families of function
4. Piecewise functions
5. Composition of functions

C. Exponential and logarithmic functions

1. Exponential growth and decay
2. Inverse functions
3. Logarithmic functions
4. Properties of logarithms

D. Trigonometric functions

1. Graphs of basic trigonometric functions
    a. Domain and range
    b. Transformations
    c. Inverse trigonometric functions
2. Applications

Unit 2: Limits and Continuity (3 weeks) [C2]

> **C2—**The course teaches all topics associated with Functions, Graphs, and Limits; Derivatives; and Integrals as delineated in the Calculus AB Topic Outline in the *AP Calculus Course Description*.

A. Rates of change

B. Limits at a point

1. Properties of limits
2. Two-sided
3. One-sided

C. Limits involving infinity

1. Asymptotic behavior
2. End behavior
3. Properties of limits
4. Visualizing limits

D. Continuity

1. Continuous functions
2. Discontinuous functions
    a. Removable discontinuity
    b. Jump discontinuity
    c. Infinite discontinuity

E. Instantaneous rates of change

Unit 3: The Derivative (5 weeks) [C2]

A. Definition of the derivative

B. Differentiability

1. Local linearity
2. Numeric derivatives using the calculator
3. Differentiability and continuity

C. Derivatives of algebraic functions

D. Derivative rules when combining functions

E. Applications to velocity and acceleration

F. Derivatives of trigonometric functions

G. The chain rule

H. Implicit derivatives

1. Differential method
2. y' method

I. Derivatives of inverse trigonometric functions

J. Derivatives of logarithmic and exponential functions

Unit 4: Applications of the Derivative (4 weeks) [C2]

C2—The course teaches all topics associated with Functions, Graphs, and Limits; Derivatives; and Integrals as delineated in the Calculus AB Topic Outline in the *AP Calculus Course Description*.

A. Extreme values

1. Local (relative) extrema
2. Global (absolute) extrema

B. Using the derivative

1. Mean value theorem
2. Rolle's theorem
3. Increasing and decreasing functions

C. Analysis of graphs using the first and second derivatives

1. Critical values
2. First derivative test for extrema
3. Concavity and points of inflection
4. Second derivative test for extrema

D. Optimization problems

E. Linearization models

F. Related rates

Unit 5: The Definite Integral (3 weeks) [C2]

A. Approximating areas

1. Riemann sums

2. Trapezoidal rule

3. Definite integrals

B. The Fundamental Theorem of Calculus (part 1)

C. Definite integrals and antiderivatives

1. The Average Value Theorem

D. The Fundamental Theorem of Calculus (part 2)

Unit 6: Differential Equations and Mathematical Modeling (3-4 weeks) [C2]

A. Antiderivatives

B. Integration using *u*-substitution

C. Separable differential equations

1. Growth and decay

2. Slope fields

3. General differential equations

Unit 7: Applications of Definite Integrals (3 weeks) [C2]

A. Summing rates of change

B. Particle motion

C. Areas in the plane

D. Volumes

1. Volumes of solids with known cross sections.

2. Volumes of solids of revolution

   a. Disk method

   b. Shell method

This schedule leaves 4–6 weeks for flexibility with teaching and learning time management.

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